CSE 386C METHODS OF APPLIED MATHEMATICS
List of Required Theorems

Exam 1

1. Properties of Minkowski Set (Prop. 5.2.2)
2. Properties of Minkowski Functional (Prop. 5.2.3)
3. Equivalent ways to set up a l.c.t.v.s (Prop. 5.2.4)
4. Characterization of convergence in Schwartz test space (Prop. 5.3.2)
5. Hanh-Banach Theorem (Thm. 5.4.1)
6. Bohnenblust - Sobczyk Theorem (Thm. 5.5.1)
7. Characterization of Continuous Linear Operators in Normed Spaces (Prop. 5.6.1)
8. Different formulas for the norm of a bounded linear operator (Prop. 5.6.2)
9. Completeness of $\mathcal{L}(X, Y)$ (Prop. 5.7.1)
10. Uniform Boundedness Theorem (Thm. 5.8.1)
11. Banach - Steinhaus Theorem (Thm. 5.8.2)
12. The Open Map Theorem (Thm. 5.9.1) with The Banach Theorem as corollary.

Exam 2

1. Characterization of closed operators (Prop. 5.10.2)
2. Necessary and sufficient conditions for an operator to be closed (Prop. 5.10.3)
3. Closed Graph Theorem (Thm. 5.10.1)
4. Representation Theorem for $(L^p(\Omega))'$ (Thm. 5.12.1)
5. Integral Form of Minkowski's Inequality (Prop. 5.12.1)
6. Mazur separation Theorem (Lemma 5.13.1)
7. Properties of reflexive spaces (Prop. 5.13.1)
8. Boundedness of weakly convergent sequences (Prop. 5.14.2)
9. Characterization of weakly convergent sequences (Prop. 5.14.3)
10. Weak Sequential Compactness (Thm. 5.14.1)

11. Characterization of linear continuous compact operators (Prop. 5.15.1)

12. Arzelà-Ascoli Theorem (Thm. 4.9.3)

13. Properties of the transpose of a continuous operator (Prop. 5.16.1)

14. Characterization of double orthogonal complements (Prof. 5.16.2)

15. Relation between range and orthogonal complement of the null space of the adjoint (Prop. 5.17.1)

16. Characterization of injective operators with closed range (Thm. 5.17.1)

17. Completness of qoutient Banach space (Lemma 5.17.1)

Exam 3

1. The Closed Range Theorem for Continuous Operators (Thm 5.17.3) with proof of (i) $\Rightarrow$ (ii) only.

2. Properties of the transpose for a closed operator (Prop. 5.18.1)

3. Characterization of closed operators with closed range (Prop. 5.18.2, Thm. 5.18.1)

4. The Orthogonal Decomposition Theorem (Thm. 6.2.1)

5. Riesz Representation Theorem (Thm. 6.4.1)

6. Properties of the adjoint operators (Prop. 6.5.1, 6.5.2)

7. Babuška-Nečas Theorem (Thm 6.6.1)

8. Lax-Milgram Theorem (Thm 6.6.2)

Hints for the Final

1. Definition and fundamental properties of normal operators: Prop. 6.5.3, Cor. 6.5.1, and Exercises 6.5.1-9.

2. Fundamentals of Spectral Theory: Section 6.8-9 and Exercises there.

3. Spectral Theory for Compact Operators: Section 6.10.

4. Variational Problems evolving around Babuška-Nečas Theorem and discussed examples in 1D and 2D (diffusion-convection-reaction problem).

5. Exercises 6.3.1-5.

6. Exercises 6.2.1-4.

7. Exercises 6.1.1-7.