EM311M - Dynamics
Exam 1
Wednesday, Feb 16, 2011, WEL 3.502, 6:00-8:00 pm

1. A particle is moving in a straight line with a an acceleration $a(t) = c(t-2)$, where c is a positive constant. At the time $t_0 = 2$, the corresponding position and velocity are x_0 and v_0, respectively. Derive the formulas for position $x(t)$ and velocity $v(t)$ at any time t (5 points)

$$\ddot{x} = c(t-2) \Rightarrow \int^{t} \ddot{x}\,dt = \int_2^t c(t-2)\,dt$$

$$\therefore \dot{x}(t) - \underbrace{\dot{x}(2)}_{v_0} = c\left.\frac{(t-2)^2}{2}\right|_2^t = \frac{c}{2}(t-2)^2$$

$$\therefore \boxed{\dot{x}(t) = v_0 + \frac{c}{2}(t-2)^2} \Big/ \int_2^t dt$$

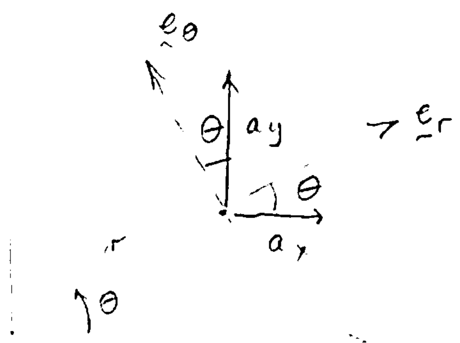
(2)

$$x(t) - \overset{x_0}{x(2)} = \left[v_0 t + \frac{c}{6}(t-2)^3\right]_2^t$$

$$\therefore \boxed{x(t) = x_0 + v_0(t-2) + \frac{c}{6}(t-2)^3}$$

(3)

2. At point $r = 2$m, $\theta = 30°$, the Cartesian components of an acceleration vector of particle at the point are $(2, 3)$ [m/s^2]. Calculate the polar components a_r, a_θ of the acceleration vector. (5 points)



$$a_r = a_x \cos\theta + a_y \sin\theta$$
$$= 2\cos 30° + 3\sin 30°$$
$$= 3.232 \left[\frac{m}{s^2}\right]$$

$$a_\theta = -a_x \sin\theta + a_y \cos\theta$$
$$= -2\sin 30° + 3\cos 30°$$
$$= 1.598 \left[\frac{m}{s^2}\right]$$

3. Can a particle moving on a curvilinear path have a zero acceleration (vector)? Explain. (5 points)

Not along a portion of path!
Since particle is moving, its velocity component $v \neq 0$; since path is curvilinear, $\rho \neq 0$, so $a_n = \frac{v^2}{\rho} \neq 0$

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But, the acceleration is equal zero at some special points (see the separate page for an example of such a situation)

4. Derive the formulas for acceleration vector components a_r and a_θ, a_z in the cylindrical system of coordinates (5 points)

$$\underset{\sim}{r} = r\underset{\sim}{e}_r + z\underset{\sim}{e}_z$$

$$\dot{\underset{\sim}{r}} = \dot{r}\underset{\sim}{e}_r + r\frac{d\underset{\sim}{e}_r}{d\theta}\dot{\theta} + \dot{z}\underset{\sim}{e}_z$$

$$= \underbrace{\dot{r}}_{v_r}\underset{\sim}{e}_r + \underbrace{r\dot{\theta}}_{v_\theta}\underset{\sim}{e}_\theta + \underbrace{\dot{z}}_{v_z}\underset{\sim}{e}_z$$

$$\underset{\sim}{e}_r = (\cos\theta, \sin\theta)$$

$$\frac{d\underset{\sim}{e}_r}{d\theta} = (-\sin\theta, \cos\theta) = \underset{\sim}{e}_\theta$$

$$\frac{d\underset{\sim}{e}_\theta}{d\theta} = (-\cos\theta, -\sin\theta) = -\underset{\sim}{e}_r$$

$$\ddot{\underset{\sim}{r}} = \ddot{r}\underset{\sim}{e}_r + \dot{r}\underbrace{\frac{d\underset{\sim}{e}_r}{d\theta}}_{\underset{\sim}{e}_\theta}\dot{\theta} + \dot{r}\dot{\theta}\underset{\sim}{e}_\theta + r\ddot{\theta}\underset{\sim}{e}_\theta + r\dot{\theta}\underbrace{\frac{d\underset{\sim}{e}_\theta}{d\theta}}_{-\underset{\sim}{e}_r}\dot{\theta} + \ddot{z}\underset{\sim}{e}_z$$

$$= \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r}\underset{\sim}{e}_r + \underbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})}_{a_\theta}\underset{\sim}{e}_\theta + \underbrace{\ddot{z}}_{a_z}\underset{\sim}{e}_z$$

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5. A particle moves along a parabola $y^2 = x$ with a constant speed v. Determine the Cartesian components of the velocity vector as a function of coordinate y. (5 points)

$$y^2 = x \qquad \dot{x}^2 + \dot{y}^2 = v^2$$

$$2y\dot{y} = \dot{x} \quad \Rightarrow \quad (2y\dot{y})^2 + \dot{y}^2 = v^2$$

$$\dot{y}^2(4y^2+1) = v^2 \Rightarrow \dot{y} = \pm\frac{v}{\sqrt{1+4y^2}}$$

$$\therefore \dot{x} = \pm\frac{2yv}{\sqrt{1+4y^2}}$$

the signs will depend upon the position and direction of motion

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Problem 3

Consider a particle moving along path $y = x^3$.

Then :

$$\dot{y} = 3x^2\dot{x}$$
$$\ddot{y} = 6x\dot{x}^2 + 3x^2\ddot{x}$$

Assume that at point $x=y=0$, $\dot{x} \neq 0$ but $\ddot{x} = 0$

Then at $(0,0)$,

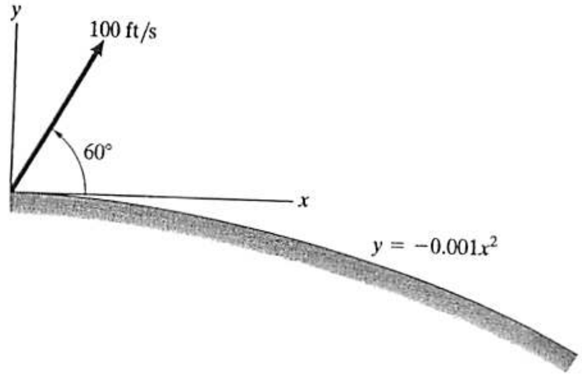
$$\underset{\sim}{v} = (\dot{x}, \dot{y}) = (\dot{x}, 0) \neq \underset{\sim}{0} \quad \text{(the particle is moving)}$$

but

$$\underset{\sim}{a} = (\ddot{x}, \ddot{y}) = \underset{\sim}{0}$$

$\#$

6. A projectile is launched at 100 ft/s at 60^o above the horizontal. The surface on which it lands is described by the equation shown. Determine the x coordinate of the point of impact. (25 points)



$\ddot{x} = 0$

5 $\dot{x} = 100 \cos 60° = 50$ [ft/s]

$x = 50t$ [ft]

$\ddot{y} = -32.2$ [ft/s²]

5 $\dot{y} = -32.2t + 100 \sin 60° = -32.2t + 86.60$ [ft/s]

$y = -16.1t^2 + 86.6t$ [ft]

At the point of impact;

$y = -0.001x^2$ 5

Substituting,

$-16.1t^2 + 86.6t = -0.001(50t)^2 = -2.5t^2$

$-13.6t^2 + 86.6t = 0$

$t = 0$ or $\boxed{t = 6.368 \text{ [s]}}$ 5

initial point

So $x = 50 \cdot 6.368 = \underline{318.4}$ [ft]

$y = -101.4$ [ft]

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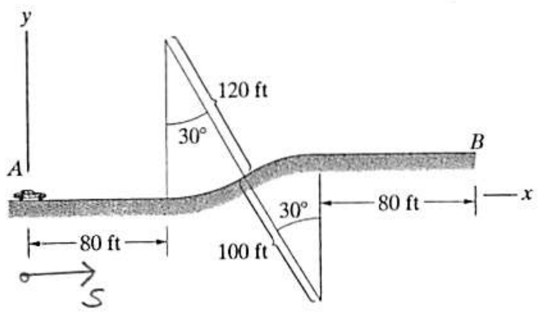
7. The car increases its speed at a constant rate from 40 mi/h at A to 60 mi/h at B. What is the magnitude of its acceleration 2s after it passes point A? (25 points)



$$V_A = 40\ \frac{mi}{h} = 40 \cdot \frac{5280}{3600} = 58.67\ \left[\frac{ft}{s}\right]$$

$$V_B = 60\ \frac{mi}{h} = 88\ \left[\frac{ft}{s}\right]$$

$$S_A = 0, \quad S_B = 80 + \overbrace{\frac{30}{360} \cdot 2\pi \cdot 120}^{62.83} + \frac{30}{360} \cdot 2\pi \cdot 100 + 80$$

$$= 275.2\ [ft]$$

(5)

$$\int_{S_A}^{S_B} a_t\, ds = \int_{V_A}^{V_B} v\, dv$$

$$a_t \cdot 275.2 = \frac{1}{2}(V_B^2 - V_A^2) = \frac{1}{2}(88^2 - 58.67^2)$$

$$\underline{a_t = 7.82\ \left[\frac{ft}{s}\right]}$$

(5)

$$\dot{v} = 7.82 \ / \int_0^2 dt$$

$$\underset{\overset{\parallel}{58.67}}{v(2) - v(0)} = 7.82 \cdot 2; \quad \Rightarrow \underline{v(2) = 74.31\ \left[\frac{ft}{s}\right]}$$

(5)

$$v(t) = 58.67 + 7.82\, t \ / \int_0^2 dt ,$$

(5)

$$s(2) = \left(58.67t + 7.82\frac{t^2}{2}\right)\Big|_0^2 = \underline{132.98} < 80 + 62.83$$

At $t = 2s$, the car will be on the first circular segment

so:

$$a_n = \frac{v^2}{\rho} = \frac{74.31^2}{120} = 46.02\ [ft/s^2]$$

(5)

$$a = \sqrt{a_n^2 + a_t^2} = \sqrt{46.02^2 + 7.82^2} = \underline{46.67\ \left[\frac{ft}{s^2}\right]}$$

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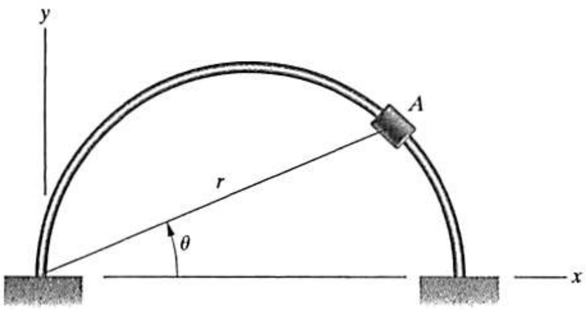
8. The collar A slides on the circular bar. The radial position of A (in meters) is given as a function of θ by $r = 2\cos\theta$. At the instant shown $\theta = 30^o$, $\dot{\theta} = 4$[rad/s], and $\ddot{\theta} = 0$. Determine the acceleration of A in polar coordinates. (25 points)



$r = 2\cos\theta = 1.73\ [ft]$

$\dot{r} = -2\sin\theta \cdot \dot{\theta} = -2\sin 30^\circ \cdot 4 = -4\ [\frac{ft}{s}]$

$\ddot{r} = -2\cos\theta \cdot \dot{\theta}^2 - \cancel{2\sin\theta \cdot \ddot{\theta}}\ 0$

$= -2\cos 30^\circ \cdot 16 = -27.71\ [\frac{ft}{s^2}]$

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$a_r = \ddot{r} - r\dot{\theta}^2$

$= -27.71 - 1.73 \cdot 16 = -55.4\ [ft/s^2]$

$a_\theta = \cancel{r\ddot{\theta}}^{\,0} + 2\dot{r}\dot{\theta} = 2(-4)\cdot 4 = -32\ [ft/s^2]$

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